Sensitivity Analysis Methods for Coupled Acoustic-Structural Systems Part II: Direct Frequency Response and Its Sensitivities

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As a continuation of the previous work, this paper deals with an investigation into the direct frequency response of a coupled acoustic-structural system and its sensitivity. If one tries to determine the direct frequency response of a coupled system using a conventional analytical technique, it may result in poor calculation efficiency for certain systems. An iteration method is proposed here that does not employ an inverse matrix (or triangular resolution) of the impedance matrix of coupled system. An algorithm is then derived for calculating the corresponding frequency response sensitivity. It is demonstrated that the use of this algorithm reduces the calculations and makes it possible to determine the frequency response and sensitivity with good accuracy.

Nomenclature

B	$= -Z_1^{-1}Z_2$
B_{as}	$= -Z_{aa}^{-1}Z_{as}$
B_{sa}	$= -Z_{ss}^{-1}Z_{sa}$
$\tilde{B}_{as}$	$= -\Omega_a^{-1}\tilde{Z}^T$
$\tilde{B}_{sa}$	$= -\Omega_s^{-1}\tilde{Z}_{sa}$
D_{ss}	$= -Z_{ss}^{-1}Z'_{ss}$
D_{aa}	$= -Z_{aa}^{-1}Z'_{aa}$
D_{sa}	$= -Z_{ss}^{-1}Z'_{sa}$
D_{as}	$= -Z_{aa}^{-1}Z'_{as}$
$\tilde{D}_{ss}$	$= -\Omega_s^{-1}\Psi_s^T Z'_{ss}\Psi_s$
F	= amplitude of f
F_s	= amplitude of f_s
f	$= \{f_s^T, 0\}^T$
f_s	= excitation force vector acting on structure
g	= constraint function
I	= unit matrix
K	= stiffness matrix of coupled system
K_{aa}	= stiffness matrix of sound field
K_{ss}	= stiffness matrix of structure
K_{sa}	= coupling term matrix
M	= mass matrix of coupled system
M_{aa}	= mass matrix of sound field
M_{ss}	= mass matrix of structure
M_{as}	$= -K_{sa}^T$
p	= sound pressure level (SPL)
Q	= modal coordinate vector
Q_a	= modal coordinate vector of sound field
Q_s	= modal coordinate vector of structure
S	$= -\Omega^{-1}\bar{\Phi}^T Z'\Phi$
U	= direct frequency response (DFR) of coupled system
U_a	= DFR for u_a
U_s	= DFR for u_s
U_n	= peak response at resonant frequency ω_n
U_{n1}	= response of undamped system at frequency ω_{n1}
U_{n2}	= response of undamped system at frequency ω_{n2}
$\boldsymbol{u}$	= nodal coordinate vector of coupled system
$\boldsymbol{u}_a$	= sound pressure vector of interior sound field
$\boldsymbol{u}_s$	= nodal displacement vector of structure
X	= adjoint vector
X_a	= component of X relative to $\boldsymbol{u}_a$
X_s	= component of X relative to $\boldsymbol{u}_s$
Z	= impedance matrix of coupled system
Z_1	= lower block triangular matrix of Z, see Eq. (8)
Z_2	= upper block triangular matrix of Z, see Eq. (8)
Z_{aa}	= impedance matrix of sound field
Z_{ss}	= impedance matrix of structure
Z_{as}	$= -\omega^2 M_{as}$
Z_{sa}	$= K_{sa}$
$\tilde{Z}_{sa}$	$= \Psi_s^T K_{sa}\Psi_a$
α	= design variable
β_n	= factor of same order as ν_n
Λ	= diag $\{\lambda_i\}$
Λ_a	= diag $\{\lambda_{ai}\}$
Λ_s	= diag $\{\lambda_{si}\}$
λ_i	= ith eigenvalue of coupled system
λ_{ai}	= ith eigenvalue of acoustic system
λ_{si}	= ith eigenvalue of structural system
μ	= given shift
ν_n	= loss factor relative to ω_n
Φ	$= [\phi_1, \phi_2, \ldots, \phi_n]$
Φ_a	= submatrix of Φ relative to $\boldsymbol{u}_a$
Φ_s	= submatrix of Φ relative to $\boldsymbol{u}_s$
ϕ_i	= ith eigenvector of coupled system
$\bar{\Phi}$	$= [\bar{\phi}_1, \phi_2, \ldots, \bar{\phi}_n]$
$\bar{\phi}_i$	= ith left eigenvector of coupled system
Ψ_a	= eigenvector matrix of acoustic system
Ψ_s	= eigenvector matrix of structural system
Ω^{-1}	= diag $\{1/(\lambda_i - \omega^2)\}$
Ω_a^{-1}	= diag $\{\omega^2/(\lambda_{ai} - \omega^2)\}$
Ω_s^{-1}	= diag $\{1/(\lambda_{si} - \omega^2)\}$
ω	= excitation frequency
ω_n	= resonant frequency
ω_{n1}	$= (1 - \frac{1}{2}\beta_n)\omega_n$
ω_{n2}	$= (1 + \frac{1}{2}\beta_n)\omega_n$
$(\,)'$	= partial derivative of ( ) with respect to design variable α
$(\,)^{(n)}$	= nth approximate solution of ( )
diag { }	= diagonal matrix

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Introduction

THE first report[1] by the authors presented an analytical method for determining the sensitivities of the eigenvalue and eigenvector of coupled acoustic-structural systems. As a continuation of the previous work, this paper deals with an investigation into the direct frequency response (DFR) of coupled systems and proposes a technique for analyzing their sensitivities.

The impedance matrix that has been treated in conventional structural analysis is symmetrical and has a certain bandwidth. As well known, these characteristics can be used to good advantage to increase the calculation speed for a DFR analysis. By contrast, in the case of a coupled acoustic-structural system, the impedance matrix is asymmetrical and virtually loses its bandwidth characteristic. Consequently, if one tries to determine the DFR using a conventional analytical technique for this purpose, it may result in poor calculation efficiency. To overcome this problem, an iteration method is proposed here, which deals with only the impedance matrices of the uncoupled structural and acoustic systems instead of employing an inverse matrix (or triangular resolution) of the impedance matrix of the entire coupled system. Then, an algorithm is derived for calculating the corresponding frequency response sensitivity. It will be demonstrated that the use of this algorithm greatly reduces the calculations and makes it possible to determine the frequency response and sensitivity with good accuracy.

Finally, the calculations using the eigenmodes of the structural system and acoustic system or those of the coupled system will be presented, and the sound pressure level (SPL) sensitivity in relation to the design variables will be examined. The validity of the proposed method will be verified by applying it to a coupled acoustic-box structural system and to a coupled acoustic-vehicle system.

Direct Frequency Response of a Coupled Acoustic-Structural System

Previous Method

This discussion will examine the following finite element equations for treating a coupled acoustic-structural system.[1]

$$M\ddot{u} + Ku = f \tag{1}$$

where

$$K = \begin{bmatrix} K_{ss} & K_{sa} \\ 0 & K_{aa} \end{bmatrix}, \qquad M = \begin{bmatrix} M_{ss} & 0 \\ M_{as} & M_{aa} \end{bmatrix}$$

$$u = \begin{Bmatrix} u_s \\ u_a \end{Bmatrix}, \qquad f = \begin{Bmatrix} f_s \\ 0 \end{Bmatrix} \tag{2}$$

Here, u_s is the displacement vector of the structure, u_a the sound pressure vector, f_s the excitation force acting on the structure, M_{ss} and K_{ss} the mass and stiffness matrices of the structure, M_{aa} and K_{aa} the mass and stiffness matrices of the sound field, and M_{as} and K_{sa} the coupled term matrices, $K_{sa} = -M_{as}^T$. The subscripts s and a denote the structural system and sound field system, respectively. The notation $(\ddot{\ })$ indicates the second derivative of u with respect to time.

Letting $f_s = F_s e^{i\omega t}$ represent the excitation force and $u = Ue^{i\omega t}$ the response, the following expression for the frequency response is obtained from Eq. (1):

$$ZU = F \tag{3}$$

where Z is the impedance matrix of the coupled system,

$$Z = \begin{bmatrix} K_{ss} - \omega^2 M_{ss} & K_{sa} \\ -\omega^2 M_{as} & K_{aa} - \omega^2 M_{aa} \end{bmatrix} \tag{4}$$

and,

$$U = \begin{Bmatrix} U_s \\ U_a \end{Bmatrix}, \qquad F = \begin{Bmatrix} F_s \\ 0 \end{Bmatrix} \tag{5}$$

Also, U_s and U_a stand for the components of DFR relative to the structural and acoustic coordinates, respectively, F_s the amplitude of the excitation force f_s, and ω the frequency of the excitation force.

By solving Eq. (3) directly for a certain given frequency ω, the DFR is obtained as

$$U = Z^{-1}F \tag{6}$$

This expression is typical of the conventional approach to DFR analysis. Whereas Eq. (6) does yield a strict solution, it has been pointed out that the calculation volume becomes inordinately large, especially when an attempt is made to find the response for many frequencies. The impedance matrix Z that has been treated in conventional structural analysis is symmetrical and possesses a certain bandwidth. This characteristic can be utilized to good advantage to increase the calculation speed to a certain extent. By contrast, in the case of a coupled acoustic-structural system, the impedance matrix Z is asymmetrical, as seen in Eq. (4) and virtually loses its bandwidth characteristic. Consequently, from the standpoint of the calculation volume, the DFR method becomes even more disadvantageous. To overcome this drawback, a simplified method for calculating the DFR of coupled systems will be considered in the following section.

Iteration Method for Calculating Direct Frequency Response

First, we will separate the impedance matrix Z in Eq. (4) into the lower block triangular matrix Z_1 and the remaining matrix Z_2 as shown here

$$Z_1 = \begin{bmatrix} K_{ss} - \omega^2 M_{ss} & 0 \\ -\omega^2 M_{as} & K_{aa} - \omega^2 M_{aa} \end{bmatrix}, \qquad Z_2 = \begin{bmatrix} 0 & K_{sa} \\ 0 & 0 \end{bmatrix} \tag{7}$$

where

$$Z = Z_1 + Z_2 \tag{8}$$

Then, Eq. (3) can be rewritten as

$$Z_1 U = F - Z_2 U \tag{9}$$

Assuming that $U^{(0)} = Z_1^{-1}F$, the following iteration expression is obtained from Eq. (9):

$$U^{(n)} = U^{(0)} + BU^{(n-1)} \qquad (n = 1, 2, \ldots) \tag{10}$$

where $U^{(n)}$ is the nth approximate solution for U, and

$$B = -Z_1^{-1}Z_2 \tag{11}$$

The necessary and sufficient condition for the convergence of Eq. (10) is $\|B\| < 1$ or the maximum eigenvalue of B less than 1, where $\|B\|$ is the norm of the matrix B. Because the value of $\|B\|$ in Eq. (11) is generally much smaller than 1, sufficient accuracy can be obtained by performing a small number of iterations on Eq. (10). However, since $\|B\|$ is a function of the frequency ω, it can become larger than one when ω coincides with the natural frequency of the structure or of the sound field. As a result, Eq. (10) will not converge. A frequency of this type is near the resonant frequency of the coupled system and must be avoided in carrying out the aforementioned iteration calculations.

It should be noted that since the damping terms have been omitted from elemental Eq. (1), the peak response at a resonant frequency can not be obtained using the aforementioned formulations. However, this incompletion can be overcome by using Ewins's method[2] as follows. Let ω_n be a resonant frequency of the system, and ν_n the corresponding loss factor. If U_{n1} and U_{n2}, the undamped responses at frequencies given by

$$\omega_{n1} = (1 - \tfrac{1}{2}\beta_n)\omega_n, \qquad \omega_{n2} = (1 + \tfrac{1}{2}\beta_n)\omega_n \tag{12}$$

have been obtained, the peak response U_n can be estimated from

$$|U_n| = \beta_n/\nu_n\sqrt{\tfrac{1}{2}(U_{n1}^2 + U_{n2}^2)} \tag{13}$$

where β_n is a factor of the same order as ν_n.

Equation (10) can be divided into the structural response U_s and the acoustic response U_a and expressed as follows to yield the calculation algorithm

$$U_s^{(0)} = Z_{ss}^{-1}F_s, \qquad U_a^{(0)} = B_{as}U_s^{(0)} \tag{14}$$

$$U_s^{(n)} = U_s^{(0)} + B_{sa}U_a^{(n-1)}, \qquad U_a^{(n)} = B_{as}U_s^{(n)} \qquad (n = 1,2, \ldots) \tag{15}$$

where

$$B_{as} = -Z_{aa}^{-1}Z_{as}, \qquad B_{sa} = -Z_{ss}^{-1}Z_{sa} \tag{16}$$

Here, Z_{ss} is the impedance matrix of the structure and Z_{aa} is the impedance matrix of the sound field, which are expressed as

$$Z_{ss} = K_{ss} - \omega^2 M_{ss}, \qquad Z_{aa} = K_{aa} - \omega^2 M_{aa} \tag{17}$$

and, Z_{as} and Z_{sa} are expressed as $Z_{as} = \omega^2 Z_{sa}^T = -\omega^2 M_{as}$.

Letting N_s represent the number of degrees of freedom of the structure and N_a the number of degrees of freedom of the sound field, the total number of the coupling equations in Eq. (3) is $N = N_s + N_a$. The coefficient matrix Z of this equation is asymmetrical and has virtually a full band. On the other hand, the matrices treated in the calculations in Eqs. (14) and (15) are Z_{ss} and Z_{aa}, the impedance matrices of the structure and sound field, respectively. Whereas Eq. (15) requires iterative calculations, only one computation is needed for the inverse matrices (or triangular resolution) of Z_{ss} and Z_{aa}. It is well known that, in treating the coupled equation of one higher order, $N_s + N_a$, the computation volume for solving the equations of the two lower orders, N_s and N_a, is greatly reduced as N_s and N_a become larger. Also, a conventional FEM code, such as NASTRAN, can be used to calculate Z_{ss}^{-1} and Z_{aa}^{-1}. Consequently, the symmetry and band nature of the coefficient matrices, Z_{ss} and Z_{aa}, can be used as in the past to speed up the calculations.

Direct Frequency Response Sensitivity

Direct Method Using Z^{-1}

The aim of a frequency response sensitivity analysis is to find the derivatives (sensitivities) in the frequency response relative to the given design variables. Letting α represent a design variable, the sensitivity of the frequency response U relative to α can be written as U'. In this section, we will examine a method for finding the sensitivity U' in the DFR.

Letting Z' represent the derivative of impedance matrix Z in Eq. (3) relative to the design variable α, Here, Z' is given as

$$Z' = \begin{bmatrix} K_{ss}' - \omega^2 M_{ss}' & K_{sa}' \\ -\omega^2 M_{as}' & K_{aa}' - \omega^2 M_{aa}' \end{bmatrix} = \begin{bmatrix} Z_{ss}' & Z_{sa}' \\ Z_{as}' & Z_{aa}' \end{bmatrix} \tag{18}$$

Performing a partial derivative operation on Eq. (3) results in

$$Z'U + ZU' = 0 \tag{19}$$

Then, the following expression is obtained from Eq. (19)

$$U' = -Z^{-1}Z'U \tag{20}$$

Equation (20) can be used to find the sensitivity of DFR, U', directly. It is seen in the computation in Eq. (20) that the inverse matrix Z^{-1} (or triangular resolution of Z) must be calculated. Consequently, as discussed in the prior section, it may result in poor calculation efficiency. The following section will examine a sensitivity analysis method that is based on the iteration method described in prior section and is designed to avoid the computation of Z^{-1}.

Iteration Method Without Using Z^{-1}

The calculation algorithm presented in this paper for finding the DFR is based on one type of iteration method and avoids the computation of Z^{-1}. This section will consider a corresponding sensitivity analysis method.

Substituting Eq. (8) into Eq. (20) yields the following iteration equation:

$$U'^{(n)} = U'^{(0)} + BU'^{(n-1)} \qquad (n = 1,2, \ldots) \tag{21}$$

where $U'^{(0)} = -Z_1^{-1}Z'U$ and $U'^{(n)}$ is the nth approximate solution for U'.

Equation (21) can be further divided into the frequency response of the structure and that of the sound field, which results in the following expressions:

When Z_{ss}' alone is not zero

$$U_s'^{(0)} = D_{ss}U_s, \qquad U_a'^{(0)} = B_{as}U_s'^{(0)}$$

$$U_s'^{(n)} = U_s'^{(0)} + B_{sa}U_a'^{(n-1)}$$

$$U_a'^{(n)} = B_{as}U_s'^{(n)}, \qquad (n = 1,2, \ldots) \tag{22}$$

where $D_{ss} = -Z_{ss}^{-1}Z_{ss}'$.

When Z_{aa}' alone is not zero

$$U_s'^{(0)} = 0, \qquad U_a'^{(0)} = D_{aa}U_a$$

$$U_s'^{(n)} = B_{sa}U_a'^{(n-1)}$$

$$U_a'^{(n)} = U_a'^{(0)} + B_{as}U_s'^{(n)}, \qquad (n = 1,2, \ldots) \tag{23}$$

where $D_{aa} = -Z_{aa}^{-1}Z_{aa}'$.

When Z_{as}' and Z_{sa}' alone are not zero

$$U_s'^{(0)} = D_{sa}U_a, \qquad U_a'^{(0)} = D_{as}U_s + B_{as}U_s'^{(0)}$$

$$U_s'^{(n)} = U_s'^{(0)} + B_{as}U_a'^{(n-1)}$$

$$U_a'^{(n)} = B_{sa}(U_s'^{(n)} - U_s'^{(0)}), \qquad (n = 1,2, \ldots) \tag{24}$$

where $D_{sa} = -Z_{ss}^{-1}Z_{sa}'$ and $D_{as} = -Z_{aa}^{-1}Z_{as}'$.

When either Eq. (22), Eq. (23), or Eq. (24) is applied, the only computation required is that for the inverse matrix (or triangular resolution) of the impedance matrices (Z_{ss} and Z_{aa}) of the uncoupled system. Consequently, the computations can be substantially reduced as described in the previous section.

The sensitivity of the peak response at a resonant frequency can not be obtained by the aforementioned formulations. However, this incompletion can be overcome by expanding Ewins's method, which was mentioned in the previous section, to the peak sensitivity calculation.

It should be noted that the iteration algorithm described earlier needs to be performed once for each design variable α, so it becomes costly when the number of design variable is large. However, this problem can be overcome by using the adjoint variable method. (See, for example, Ref. 5.)

Let us suppose that a (output) function, g, is represented as[5]

$$g = g(U, \alpha) \tag{25}$$

Then, using the iteration method proposed in this paper, a computational algorithm for solving the adjoint vector, X, can be obtained as

$$X^{(n)} = X^{(0)} + BX^{(n-1)} \qquad (n = 1,2, \ldots) \tag{26}$$

where $X^{(n)}$ is the nth approximate solution for the adjoint vector X , and $X^{(0)} = Z_1^{-1}(\partial g/\partial U)$.

Further, a divided form of Eq. (26) relative to the structural and acoustic coordinates is given as

$$X_a^{(n)} = X_a^{(*)} + B_{as}X_s^{(n)}$$

$$X_s^{(n+1)} = X_s^{(0)} + B_{sa}X_a^{(n)} \qquad (n = 0,1,\ldots) \tag{27}$$

where $X_a^{(n)}$ and $X_s^{(n)}$ are the components of $X^{(n)}$ relative to u_a and u_s, respectively, and

$$X_a^{(*)} = Z_{aa}^{-1}\frac{\partial g}{\partial U_a}, \qquad X_s^{(0)} = Z_{ss}^{-1}\frac{\partial g}{\partial U_s} \tag{28}$$

Consequently, the sensitivity of g can be obtained as

$$g' = \frac{\partial g}{\partial \alpha} - X^TZ'U \tag{29}$$

The adjoint variable method requires the performance of iteration calculation only once for each function g. Therefore, the adjoint variable method is more efficient when the number of functions is smaller than the number of design variables.

Calculations Using Eigenmodes

Let us assume that the eigenmodes of the structural system and acoustic system or those of the coupled system have already been found. Then, those eigenmodes can be used in performing the calculations by converting the equations presented in prior sections to modal coordinate systems. This section will examine calculation procedures that employ eigenmodes.

Calculations Using Eigenmodes of Coupled Systems

We will let $\Lambda = \text{diag}\ \{\lambda_i\}$ represent the eigenvalue matrix of a coupled system and $\Phi = [\phi_1, \phi_2, \ldots, \phi_n]$ the right eigenvector matrix, and, for the sake of simplicity, assume that the system does not have any zero eigenvalues and repeated eigenvalues. Then, using the orthogonality and normalization conditions of coupled system presented in Ref. 1, the following expanded equation is obtained for Z^{-1} (see appendix)

$$Z^{-1} = \Phi\Omega^{-1}\bar{\Phi}^T \tag{30}$$

where

$$\bar{\Phi}^T = \{\Phi_s^T,\ \Phi_a^T\Lambda^{-1}\}, \qquad \Omega^{-1} = \text{diag}\left\{\frac{1}{\lambda_i - \omega^2}\right\}$$

and Φ_s and Φ_a are the submatrices of the eigenvector matrix Φ relative to the structural and acoustic coordinates, respectively.

Suppose the frequency response U can be expanded to the eigenvectors of coupled system as

$$U = \Phi Q \tag{31}$$

then substituting Eqs. (30) and (31) into Eq. (6), it results $Q = \Omega^{-1}\bar{\Phi}^TF$.

Suppose the sensitivity of DFR, U' can also be expanded to the eigenvectors, which results in

$$U' = \Phi Q' \tag{32}$$

Then, by substituting Eqs. (31) and (32) into Eq. (20) and using Eq. (30) we can obtain

$$Q' = SQ \tag{33}$$

where

$$S = -\Omega^{-1}\bar{\Phi}^TZ'\Phi \tag{34}$$

Since Eqs. (31) and (32) can be calculated using only the lower-order modes, the higher-order modes can be omitted, thereby reducing the computations.

Calculations Using Uncoupled System Modes

We will let $\Lambda_s = \text{diag}\{\lambda_{si}\}$ and $\Psi_s = [\psi_{s1}, \psi_{s2}, \ldots, \psi_{sn_s}]$ represent the eigenpair matrices of the structural system and $\Lambda_a = \text{diag}\{\lambda_{ai}\}$ and $\Psi_a = [\psi_{a1}, \psi_{a2}, \ldots, \psi_{an_a}]$ the eigenpair matrices of the acoustic system. Then, the DFR components, U_s and U_a, can be expanded to Ψ_s and Ψ_a, respectively. This results in the expressions

$$U_s = \Psi_sQ_s, \qquad U_a = \Psi_aQ_a \tag{35}$$

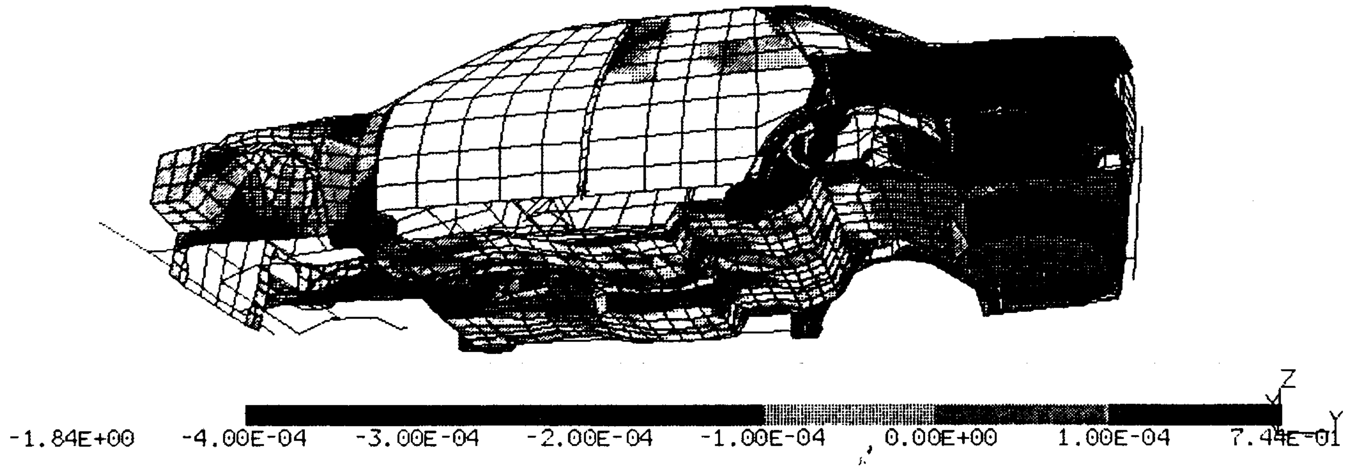

Fig. 1 Distribution of DFR sensitivities for a coupled acoustic-vehicle model (ω = 30 Hz).

Table 1 Convergence of SPL obtained with iteration method

Approx. solution	5 Hz	10 Hz	19.60 Hz	25 Hz	80 Hz
$p^{(0)}$	47.673	42.358	73.548	61.876	66.221
$p^{(1)}$	46.373	40.981	79.581	60.610	64.124
$p^{(2)}$	46.569	41.224	85.122	60.820	64.664
$p^{(3)}$	46.541	41.191	90.428	60.788	64.542
$p^{(4)}$	46.545	41.190	95.610	60.793	64.570
$p^{(5)}$	46.545	41.190	100.72	60.792	64.565
Exact	46.545	41.190	76.07	60.792	64.565

Substituting Eq. (35) into the Eqs. (14) and (15) used to calculate the DFR yields

$$Q_s^{(0)} = \Omega_s^{-1}\Psi_s^T F_s, \qquad Q_a^{(0)} = \tilde{B}_{as}Q_s^{(0)}$$

$$Q_s^{(n)} = Q_s^{(0)} + \tilde{B}_{sa}Q_a^{(n-1)}$$

$$Q_a^{(n)} = \tilde{B}_{as}Q_s^{(n)}, \qquad (n = 1,2, \ldots) \tag{36}$$

where

$$\tilde{B}_{sa} = -\Omega_s^{-1}\tilde{Z}_{sa}, \qquad \tilde{B}_{as} = -\Omega_a^{-1}\tilde{Z}_{sa}^T \tag{37}$$

$$\Omega_s^{-1} = \text{diag}\left\{\frac{1}{\lambda_{si} - \omega^2}\right\}, \qquad \Omega_a^{-1} = \text{diag}\left\{\frac{\omega^2}{\lambda_{ai} - \omega^2}\right\},$$

$$\tilde{Z}_{sa} = \Psi_s^T K_{sa}\Psi_a \tag{38}$$

Similarly, the DFR sensitivity components, U_s' and U_a', can be expanded to Ψ_s and Ψ_a, respectively, as

$$U_s' = \Psi_s Q_s', \qquad U_a' = \Psi_a Q_a' \tag{39}$$

Substituting Eqs. (35) and (39) into the sensitivity calculation formulation, i.e., Eq. (22) or (23) or (24), we can obtain Q_s' and Q_a'. For instance, substituting Eqs. (35) and (39) into Eq. (22) yields

$$Q_s'^{(0)} = \tilde{D}_{ss}Q_s, \qquad Q_a'^{(0)} = \tilde{B}_{as}Q_s'^{(0)}$$

$$Q_s'^{(n)} = Q_s'^{(0)} + \tilde{B}_{sa}Q_a'^{(n-1)}$$

$$Q_a'^{(n)} = \tilde{B}_{as}Q_s'^{(n)}, \qquad (n = 1,2, \ldots) \tag{40}$$

where $\tilde{D}_{ss} = -\Omega_s^{-1}\Psi_s^T Z_{ss}'\Psi_s$.

Sensitivity in the Sound Pressure Level

The foregoing discussion has examined several methods for conducting a sensitivity analysis of the DFR. However, it is sometimes necessary to evaluate the rate of change in the SPL for an acoustic analysis. In this case, the SPL, p, can be expressed in terms of dBA as

$$p = 10\log_{10}(U/U_0)^2 \tag{41}$$

where $U_0 = 2 \times 10^{-5}$ Pa.

Letting p' represent the sensitivity of SPL relative to the design variable, α, and by differentiating Eq. (41), we obtain

$$p' = (20\log_{10}(U'/U) \tag{42}$$

If U' has already been found using the method described in prior sections, it can be substituted into Eq. (42) to calculate the SPL sensitivity.

Application Examples

Direct Frequency Response of a Coupled Acoustic-Box Model

The validity of the DFR analysis method described in this paper was verified using the coupled acoustic-box model shown in Fig. 1 of Ref. 1. The box was made of steel plates and measured 200 cm in length, 160 cm in width and 120 cm in height. The thickness of the steel plates was 0.8 cm. The structural model used in the analysis had 98 nodes and 96 quadrilateral plate elements (CQUAD4) and the sound field had 125 nodes and 64 solid elements (CHEXA).[3] For the sake of simplicity, the physical coordinates of the structure and sound field were first converted to their respective modal coordinate systems before the analysis was performed. In effect, that meant expressing the overall coupled system in terms of 61 degrees of freedom, using the lower-order natural vibration modes of the structure, which had 44, and the sound field, which had 17, including one rigid body mode. The resulting system having 61 degrees of freedom thus became the object of the analysis. To find the DFR of the system, it was assumed that the excitation point was the 40th node along the y-axis of the box and that the measurement point was the 32nd node in the sound field.

Table 1 shows the SPL convergence data produced with the iteration method presented in this paper and a comparison with the exact solution obtained with the conventional method, i.e., Eq. (6). Excluding the results for 19.60 Hz, all the other solutions show good convergence to five effective digits within six iterations. However, the solutions for the 19.60 Hz frequency did not converge. This is attributed to the fact that the convergence condition mentioned previously was not satisfied because the excitation frequency nearly coincided with the third-order natural vibration frequency of the system (i.e., 19.57 Hz). Therefore, the calculation of the peak response should be excluded from the aforementioned iteration. In making an estimation of the peak response for $\omega_n = 19.57$ Hz, the loss factor ν_n was assumed to be $\nu_n = 0.01$, and the factor used in Eq. (12) was $\beta_n = 0.02$. The peak response estimation found using Eq. (13) was 75.89 dBA with a 0.2% error comparing to the exact solution 76.07 dBA, which is found by expanding Eq. (3) to the damped system.

It is also seen in Table 1 that the initial solution, $U^{(0)}$, obtained with just one iteration provided a good approximation of the exact solution. This indicates that ignoring the upper right block, K_{sa}, of the impedance matrix in Eq. (4) has little effect on the solution. In actuality, this result can be explained from a physical standpoint as well, since acoustic vibration has little effect on structural vibration.[4]

Frequency Response Sensitivity of a Coupled Acoustic-Box Model

The design variables, $\alpha_1, \alpha_2, \alpha_3, \alpha_4$, used in the analysis were the respective plate thicknesses of the 1st ~ 10th, 11th ~ 20th, 21st ~ 30th, and 31st ~ 40th shell elements of the model presented in Fig. 1 of Ref. 1.

Table 2 shows the convergence of SPL sensitivity obtained with the iteration method presented in this paper and a comparison with the exact solution obtained with the conventional method, i.e., Eq. (20). The SPL sensitivity results in the table are for the design variable α_1. As seen in the table, all the solutions converged well within about seven iterations. A comparison of these results with the iteration results for the SPL in Table 1 indicates that both the SPL and SPL sensitivity show the same level of convergence.

Table 2 Convergence of SPL sensitivity obtained with iteration method for design variable α_1

Approx. solution	5 Hz	10 Hz	15 Hz	25 Hz	80 Hz
$p'^{(0)}$	0.60775	2.4825	−3.4012	0.42016	0.19488
$p'^{(1)}$	0.52114	2.0620	−2.3998	0.30359	0.22895
$p'^{(2)}$	0.53335	2.1313	−2.6890	0.32012	0.21805
$p'^{(3)}$	0.53163	2.1199	−2.6254	0.31757	0.22081
$p'^{(4)}$	0.53187	2.1218	−2.6296	0.31796	0.22015
$p'^{(5)}$	0.53184	2.1215	−2.6226	0.31790	0.22031
$p'^{(6)}$	0.53184	2.1215	−2.6246	0.31790	0.22027
Exact	0.53184	2.1215	−2.6242	0.31790	0.22028

Table 3 Sensitivity comparation for the different design variables

Frequency, Hz	SPL	Sensitivity			
		α_1	α_2	α_3	α_4
5.0	46.545	0.5318	0.7159	0.2511	−1.4273
10.0	41.190	2.1215	3.1207	1.6810	−2.3525
15.0	55.332	−2.6242	−3.8269	−2.7025	−0.6653
25.0	60.792	0.3179	−0.8389	0.2786	−0.6287
80.0	64.565	0.2203	−1.0794	−0.2464	−0.0294

Table 3 compares the results for the different design variables α_1, α_2, α_3, and α_4. It is possible to see in the results a change in the SPL sensitivity for each design variable relative to the given frequency.

Using a Cray X-MP/432 computer, the computation time required with the iteration method was 1.714 s, and that without iteration method, i.e., using Eqs. (6) and (20) was 3.075 s. The results show that the iteration method proposed here reduces the computational time and makes it possible to determine the DFR and its sensitivity with good accuracy.

Frequency Response Sensitivity Analysis for a Coupled Acoustic-Vehicle Model

A DFR sensitivity analysis was carried out using a coupled acoustic-vehicle model[1] in order to confirm the applicability of the analytical method presented in this paper to a noise analysis of an actual vehicle. As shown in Fig. 2 of Ref. 1, the vehicle model had a total of 2,982 nodes and 3,713 elements (including 2,100 CQUAD4 elements, 694 CTRIA3 elements, 732 CBAR elements, and 187 RBAR elements).[3] The sound field model had 1,374 nodes and 1,060 elements (794 CHEXA elements and 266 CPENTA elements).[3] The design variables used were the plate thicknesses of all CQUAD4 and CTRIA3 elements of which there were a total of 2,876. Excitation was applied in the z-axis direction of the left rear suspension mount and sound pressure measurements were made at the passenger's ear level in the front seat.

Figure 1 shows the SPL sensitivity distribution found with this analytical method when excitation frequency was 30 Hz. From the result in Fig. 1, it is possible to see the change in the frequency response sensitivity for each design variable relative to the certain frequency, and thus an optimization analysis can be performed for reducing the noise levels of the coupled system.[4]

Conclusion

As a continuation of the work begun in Ref. 1, a study was made of the DFR of a coupled acoustic-structural system and of methods for analyzing its DFR sensitivity. As the first step in this investigation it was necessary to improve the DFR analysis method conventionally applied to a uncoupled system. That had to be done because the coefficient matrixes of the equations for a coupled acoustic-structural system are asymmetrical and also have a full band. Consequently, an iteration technique was proposed for conducting a DFR analysis. It was shown that this iteration technique makes it possible to reduce the computations because it eliminates the need to calculate the inverse matrix of the coefficient matrix for the entire system.

As the next step, a DFR sensitivity analysis method was derived, based on the results of the foregoing DFR analysis, and its effectiveness was examined. It was shown that an iteration method, which does not employ the inverse matrix of the impedance matrix of the coupled system, is more efficient than the conventional direct method for finding the DFR sensitivity. Furthermore, it was shown that the iteration method makes it possible to determine the DFR sensitivity with good accuracy.

Finally, a coupled acoustic-box model and a coupled acoustic-vehicle model were used in the analysis to confirm the applicability of the proposed methods to actual noise analysis problems.

In future work, the authors intend to investigate a modal frequency response sensitivity analysis method and a sensitivity analysis method for coupled systems having repeated eigenvalues, as well as the application of these methods to the optimization of coupled acoustic-structural systems.

Appendix

Letting the right eigenvector matrix be $\Phi = [\phi_1, \phi_2, \ldots, \phi_n] = [\Phi_s^T, \Phi_a^T]^T$ and the eigenvalue matrix be $\Lambda = \mathrm{diag}\{\lambda_i\}$, then the left eigenvectors matrix, $\bar{\Phi}$, can be written as $\bar{\Phi} = [\bar{\phi}_1, \bar{\phi}_2, \ldots, \bar{\phi}_n] = [\Phi_s^T, \Phi_a^T\Lambda^{-1}]^T$. Therefore, the orthogonality and normalization conditions presented in Ref. 1 can be rewritten using matrices as

$$\bar{\Phi}^T K \Phi = \Lambda \quad \text{and} \quad \bar{\Phi}^T M \Phi = I \tag{A1}$$

where I is a unit matrix.

From Eqs. (4) and (A1)

$$Z = \bar{\Phi}^{-T}\bar{\Phi}^T(K - \omega^2 M)\Phi\Phi^{-1} = \bar{\Phi}^{-T}(\Lambda - \omega^2 I)\Phi^{-1} = \bar{\Phi}^{-T}\Omega\Phi^{-1} \tag{A2}$$

therefore,

$$Z^{-1} = (\bar{\Phi}^{-T}\Omega\bar{\Phi}^{-1})^{-1} = \Phi\Omega^{-1}\bar{\Phi}^T \tag{A3}$$

where

$$\Omega = \Lambda - \omega I = \mathrm{diag}\{\lambda_i - \omega^2\}, \qquad \Omega^{-1} = \mathrm{diag}\left\{\frac{1}{\lambda_i - \omega^2}\right\}$$

References

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